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TWO-DIMENSIONAL DAM-BREAK FLOOD-FLOW ANALYSIS FOR ORANGE COUNTY RESERVOIR[1]

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ABSTRACT: A two-dimensional dam-break model was used to predict the inundated area on an alluvial fan downslope from the Orange County Reservoir. The model is based upon a diffusion form of the continuity and momentum equations for long waves in shallow water, and the governing equation is solved by an explicit numerical scheme. In a comparison with a one-dimensional model, the two-dimensional model predicts a wider inundated area.
(KEY TERMS: dam break; mathematical model; two-dimensional flow; numerical methods.)

INTRODUCTION

The flood resulting from a possible dam failure is of concern to a hydrologist in the design of a reservoir. To evaluate the potential flood damage resulting from a dam break, the hydrologist usually attempts to predict not only the possibility and mode of a dam failure, but also the hydrograph of discharge from the break and the water-surface profile downstream from the dam. These tasks are completed in order to identify the inundated area, flood depth, and flow velocity. Each of these factors is important in estimating possible flood damages resulting from a dam break.

The evaluation of a dam break for an urban reservoir requires a particularly rigorous analysis because on one hand the damages are potentially large and on the other hand the costs of mitigation usually are high. Orange County Reservoir is an example of such an urban reservoir. The reservoir was constructed in the early 1940's and is located about 2.5 miles northeast of the city of Brea in urban Orange County, California (Figure 1). It has a storage capacity of 211 acre-ft, and it serves primarily as a regulating facility on the Orange County Feeder, which is a regional water-supply line. The reservoir also provides emergency water storage that can be used if the Orange County Feeder is shut down.

Orange County Reservoir is of the cut and fill type. It is rectangular in shape with rounded corners and sloping sides. Three sides of the reservoir are formed by an excavation into a hillside, and the fourth side is formed by an embankment that is about 50 ft high at its highest point. A 2-inch thick, reinforced gunite lining covers the bottom and sides of the reservoir to protect against erosion and seepage.

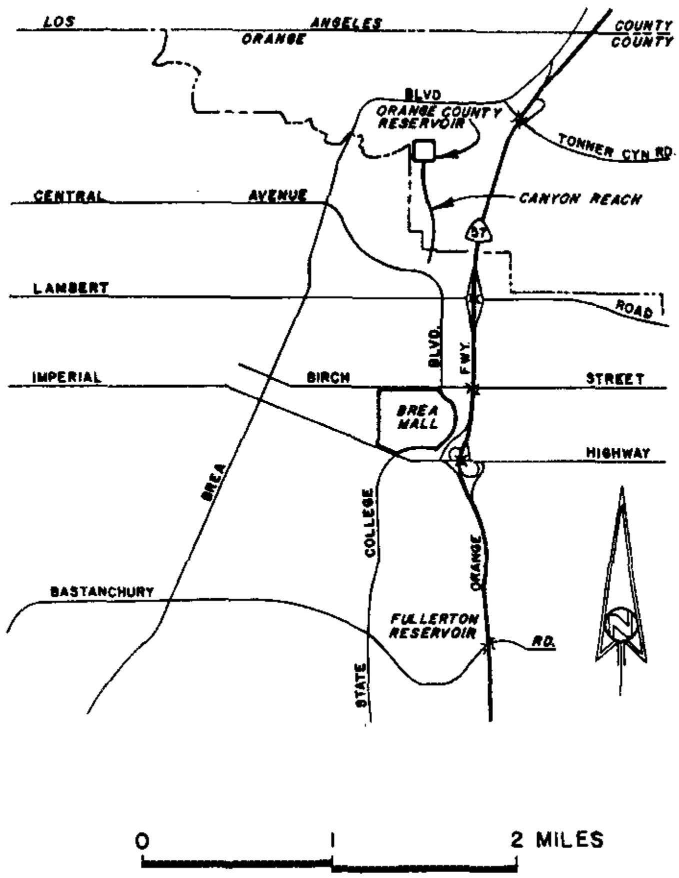

Figure 1. Location of the Study Area.

The area below Orange County Reservoir, which would potentially be flooded in the event of a dam failure, includes both a canyon reach and an alluvial fan. The canyon reach,

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which is about 2,500 ft long and has an average slope of about 300 ft per mile, is located immediately below the reservoir (Figure 1). The canyon opens into a broad alluvial fan, which has an average slope of about 100 ft per mile. Residential and commercial development occupies the alluvial fan.

A proposed regional shopping mall (Figure 1) will be located on the alluvial fan about 2.5 miles down gradient from Orange County Reservoir. The identification of the inundated area, following a possible failure of the reservoir embankment, was needed as partial fulfillment of an environmental impact report for the proposed mall. Information was particularly needed on the inundated area and flow depth in the vicinity of the proposed mall. To fulfill this need, a two-dimensional dam-break analysis was made.

TWO-DIMENSIONAL DAM-BREAK MODEL

Background

Generally a dam-break anlaysis is accomplished with a one-dimensional model of unsteady, open-channel flow. These models are based upon the numerical solution of the one-dimensional continuity and momentum equations for long waves in shallow water (Akan and Yen, 1981; Chen, 1980; Chen and Armbruster, 1980; Ponce, 1982; Rajar, 1978; Ponce and Tsivoglou, 1981; Sakkas and Strelkoff, 1973), and widely used computer algorithms are available (Land, 1980). These models are most applicable to situations where the flood discharge is mostly contained within a defined channel or floodway, but they have been applied to situations where the flood discharge spreads across a broad flood plain.

However, Hromadka, *et al.* (1985), have shown that the one-dimensional analysis may seriously underestimate the inundated area as compared with a more rigorous two-dimensional analysis. They applied both one-dimensional and two-dimensional models to the analysis of flooding in Owens Valley, California, assuming a dam-break at Crowley Lake. In this particular case, the two-dimensional analysis predicted a flood plain that was everywhere significantly wider than that predicted by one-dimensional analysis, which led to the conclusion that a two-dimensional analysis should be used for wide, flat flood plains (Hromadka, *et al.*, 1985).

The flood plain downstream from Orange County Reservoir is wide and laterally flat. Concomitantly, a two-dimensional dam-break model was used to analyze the flood following the possible failure of the embankment containing the reservoir.

Mathematical Formulations

The dam-break model is based upon the numerical solution of the two-dimensional form of the continuity and momentum equations for long waves in shallow water. The continuity equation is

$$\frac{\partial Q_x}{\partial x} \; \frac{\partial Q_y}{\partial y} + \delta \frac{\partial H}{\partial t} = 0, \tag{1}$$

where Q_x and Q_y are discharges per width δ in the x and y directions, H is water-surface elevation, x and y are Cartesian coordinates, and t is time. The momentum equations are

$$\frac{\partial Q_x}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q_x^{\,2}}{A_x}\right) + \frac{\partial}{\partial y}\left(\frac{Q_x Q_y}{A_y}\right) + g\,A_x\left(S_{fx} + \frac{\partial H}{\partial x}\right) = 0 \tag{2}$$

and

$$\frac{\partial Q_y}{\partial t} + \frac{\partial}{\partial y}\left(\frac{Q_y^{\,2}}{A_y}\right) + \frac{\partial}{\partial x}\left(\frac{Q_y Q_x}{A_x}\right) + g\,A_y\left(S_{fy} + \frac{\partial H}{\partial t}\right) = 0 \tag{3}$$

where A_x and A_y are flow areas per width δ in the x and y directions, S_{fx} and S_{fy} are the friction slopes in the x and y directions, and g is acceleration due to gravity.

For numerical solutions, the continuity and momentum equations are combined to a single diffusion equation by the following two steps.

First, the momentum equations are rearranged to obtain expressions for the local discharges Q_x and Q_y. The local and convective accelration terms of the momentum equations can be grouped together such that Equations (2) and (3) are rewritten as

$$m_z + \left(S_{fz} + \frac{\partial H}{\partial z}\right) = 0, z = x, y, \tag{4}$$

where m_z represents the first three terms of Equations (2) or (3) divided by gA_z. Assuming that the friction slope can be approximated by steady-flow conditions, Manning's equation (in English units) can be used to estimate Q_x and Q_y by the relation

$$Q_z = \frac{1.486}{n} A_z R_z^{2/3} S_{fz}^{1/2}, z = x, y, \tag{5}$$

where R_x and R_y are hydraulic radii in the x and y directions, respectively. Equation (4) can be substituted into Equation (5) with some algebraic manipulation to obtain the expression

$$Q_z = -K_z \frac{\partial H}{\partial z} - K_z m_z, z = x, y, \tag{6}$$

where

$$K_z = \frac{\frac{1.486}{n} A_z R_z^{2/3}}{\left(\frac{\partial H}{\partial z} + m_z\right)^{1/2}}, \; z = x, y. \tag{7}$$

Hromadka, *et al.* (1985), Akan and Yen (1981), and Xanthopoulos and Koutitas (1976) have shown that, for the dam-break problem, m_x and m_y are small in relation to other terms the momentum equations, and neglecting them in the analy-... produces little change in computed water-surface profiles. However, the computational effort is increased greatly when m_z is retained, but neglecting m_z results in the computationally efficient diffusion expression

$$Q_z = -K_z \frac{\partial H}{\partial z} \tag{8}$$

As the second step in developing a diffusion model, Equation (8) is substituted into the continuity equation (Equation 1) to obtain the expression

$$\frac{\partial}{\partial x}(K_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial H}{\partial y}) = \delta \frac{\partial H}{\partial t}, \tag{9}$$

where the left-hand side represents the net inflow to a local elemental volume and the right-hand side represents the rate of water-level change or storage change in the volume. If the momentum term m_z is retained in Equation (6), the equivalent expression for Equation (9) is

$$\frac{\partial}{\partial x}(K_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial H}{\partial y}) + S = \delta \frac{\partial H}{\partial t}, \tag{10}$$

where

$$S = \frac{\partial}{\partial x}(K_x m_x) + \frac{\partial}{\partial y}(K_y m_y) \tag{11}$$

and K_x and K_y are functions of m_x and m_y, respectively,

Numerical Solution

A numerical solution to Equation (9) is obtained by the integrated finite difference version of the nodal domain integration method (Hromadka and Guymon, 1982). A nodal equation is based on the local grid system shown in Figure 2. Discharge across a typical boundary of grid-block C is estimated using a linear trial function between nodal points. For a square grid of width δ,

$$Q_{x_E} = -(K_{x_E})\left(\frac{H_E - H_C}{\delta}\right) \tag{12}$$

where $_E$ is the right-hand boundary of grid-block C and

$$K_{x_E} = \frac{\frac{1.486}{n} A_x R_x^{2/3}}{\left[\frac{H_E - H_C}{\delta}\right]^{1/2}} \tag{13}$$

In Equation (13), the quantities A_x, R_x, and n are evaluated at the average water depth along the link CE. Expressions similar to Equations (12) and (13) can be developed for the other sides of grid-block C, where positive flow is W-E and S-N.

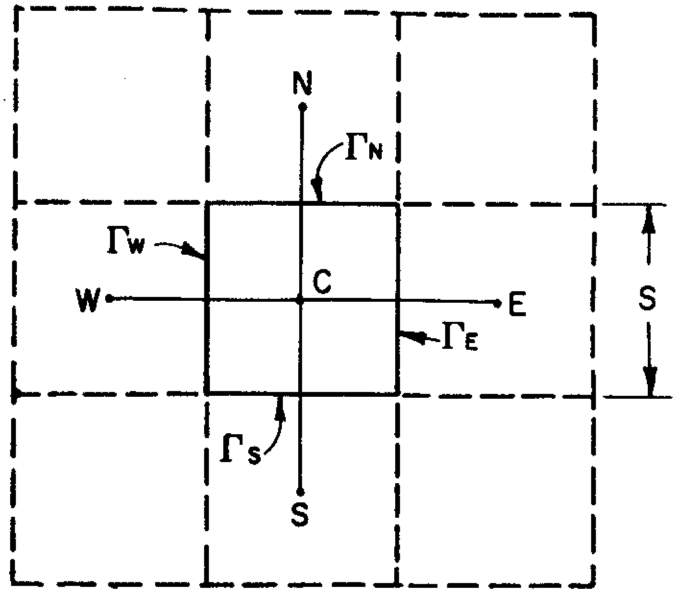

Figure 2. Local Grid System Used in Numerical Solution of Diffusion Equation.

An expression for the numerical solution of Equation (9) can be obtained by combining Equation (12) (and similar expressions for the other sides of the local grid block) with a finite-difference representation of the time derivative to obtain the approximation

$$Q_{x_E} + Q_{x_W} + Q_{y_N} + Q_{y_S} = \frac{H^{t+\Delta t} - H^t}{\Delta t}, \tag{14}$$

where Δt is the time discretization used in the approximation of the time derivative. The left-hand side of Equation (14) is evaluated at time t, and the model advances in time by explicitly solving for $H^{t+\Delta t}$ at each node (Hromadka, *et al.*, 1985).

APPLICATION TO ORANGE COUNTY RESERVOIR

Two-Dimensional Analysis

The two-dimensional diffusion model was applied to a dam-break analysis for Orange County Reservoir using the grid shown in Figure 3. The grid includes 89 square grid blocks, and the sides of the blocks are 500 ft long. The grid is used to define the geometry of the flood plain by specifying the average land-surface elevation for each grid block. A Manning's friction factor is also specifed for each grid block; these values were 0.050 for grassy areas of the alluvial fan nd 0.040 elsewhere on the alluvial fan.

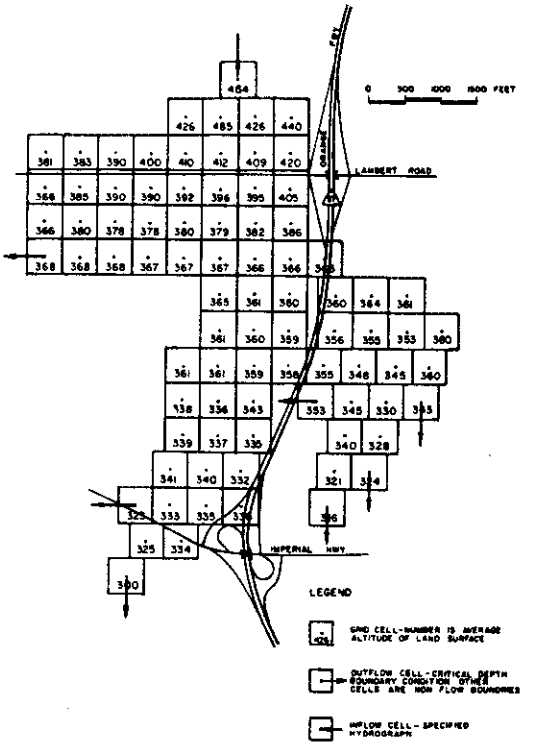

Figure 3. Map Showing Grid and Boundary Conditions Used in Dam-Break Analysis.

The grid is used also to specify the boundary conditions on the flow domain (Figure 3), which does not necessarily include the entire grid. A zero-flow boundary may be specified at the edge of the grid, or it may be established computationally on interior grid blocks if inundation does not extend to the edge of the grid at a particular location and time. Additionally, normal-depth or critical-depth boundary conditions can be specified for the boundary of the grid, if inundation extends locally to the edge of the grid, which allows water to flow off the edge of the grid at normal or critical depth. Finally, an inflow hydrograph may be specified at one or more nodal points. The inflow hydrograph is the outflow from the dam break.

One of several approaches can be used to obtain the outflow hydrograph from the dam break. The least complex is simply to assume a hydrograph of appropriate volume. A second method is to specify the geometry of the breach over time (Land, 1980). A hydrograph is developed by storage routing of the reservoir contents though the dam break, and the outflow is usually determined by assuming critical disharge through the breach. The most complex approach is to simulate the erosional and hydraulic processes that produce a breach and control the discharge through the breach as it develops (Ponce and Tsivoglou, 1981).

The hydrologic-routing method was used to develop the outflow hydrograph from a breach in the embankment of Orange County Reservoir. A triangular breach with 1-to-1 side slopes was assumed. Additionally, the lowest point of the breach was assumed to propagate downward through the embankment at a constant rate. The peak discharge through the breach is very sensitive to the time required for the breach to penetrate downward through the entire dam embankment, but the breaching period that might actually accompany a dam failure cannot be predicted. Concomitantly, to demonstrate the sensitivity of potential flood damages to different breaching periods, the dam-breach analysis was completed for breaching periods of 10, 20, and 40 minutes.

From the specification of the breach, outflow hydrographs from the reservoir were developed by storage routing of the volume of water in the reservoir through the breach based upon the storage-capacity relation shown in Figure 4. Discharge through the breach was the critical discharge for the instantaneous reservoir stage, which is given by the assumed relation

$$Q_c = \left(\frac{2g}{2}\right)^{1/2} \left(\frac{4}{5}\right)^{5/2} h^{5/2} , \qquad (15)$$

where Q_c is the critical discharge and h is the height of the reservoir water surface above the invert of the branch. Figure 5 shows the outflow hydrographs for breaching periods of 10, 20, and 40 minutes. The corresponding peak discharges are 15,600, 9,100, and 4,900 ft^3/s. In each case the volume of discharge is 211 acre-ft, which is the content of the reservoir.

Each of the hydrographs shown in Figure 5 was used separately as inflow hydrographs for the two-dimensional dam-break model. Assuming that a hydrograph translates through the canyon reach without change of shape, the hydrographs were used as input at the bottom of the canyon reach. Figure 6 shows for a 10-minute breaching period the inundated area and maximum depth of flooding, for the area down slope of Orange County Reservoir. As indicated on Figure 6, the central part of the proposed shopping mall is predicted to have about 1.2 ft of flooding. Similarly, for the 20-minute and 40-minute breaching periods, the central part of the mall is predicted to have, respectively, about 1.0 ft and 0.7 ft of flooding. Other model results for these breaching periods are shown in Figures 7 and 8.

Comparison with One-Dimensional Analysis

The two-dimensional analysis was compared with a one-dimensional analysis that was done by Metropolitan Water District of Southern California in 1973. Figure 9 shows the inundated area for both the one- and two-dimensional approaches. Everywhere, the predicted inundated area is wider for the two-dimensional analysis. Hromadka, *et al.* (1985),

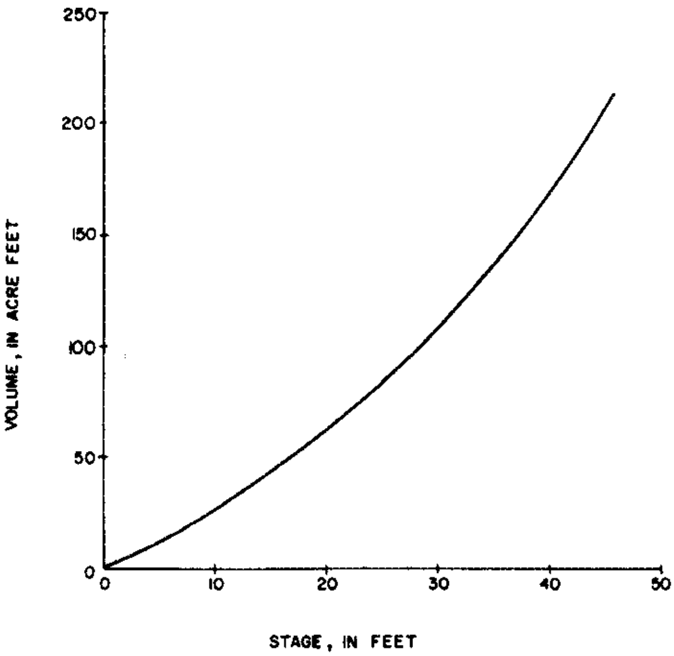

Figure 4. Relation Between Water-Surface Elevation and Storage and for Orange County Reservoir.

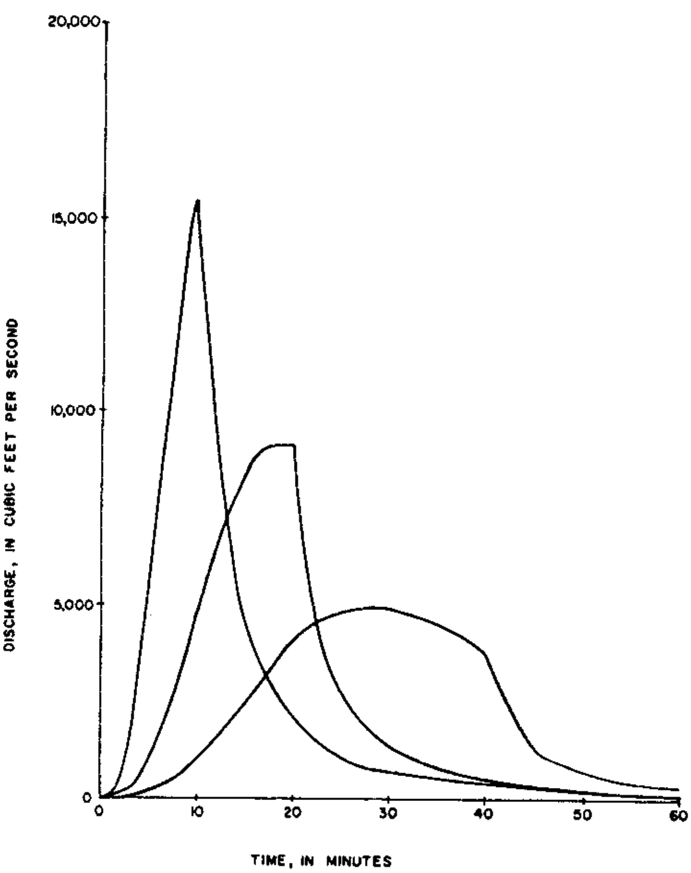

Figure 5. Outflow Hydrographs from Orange County Reservoir for Breaching Periods of 10, 20, and 40 Minutes.

obtain similar results from the comparisons of one- and two-dimensional analyses for dam-break flooding in Owens Valley, California.

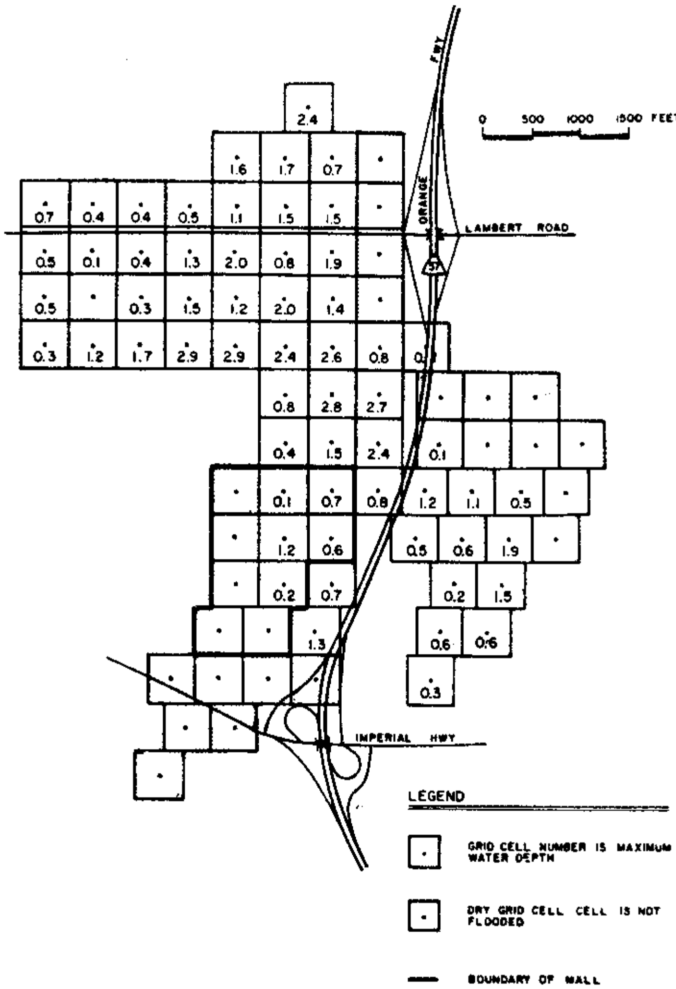

Figure 6. Maximum Flood Depths for 10-Minute Breaching Period.

The explanation for the difference follows from considerations developed from the principle of the conservation of momentum. In a one-dimensional analysis, the entire momentum of the flow is forced to be contained within the central floodway. The momentum, which is greater than actually exists, gives computed velocities which are greater than actually exist. Consequently, the computed water depths are smaller. However, in the actual field situation, lateral flows divert momentum away from the central floodway. Concomitantly, because of the relation between momentum and continuity, velocities decrease more than can be accounted for by the lost lateral discharge, and water depths in the central floodway increase. That increase further increases lateral flows. The two-dimensional model accounts for these

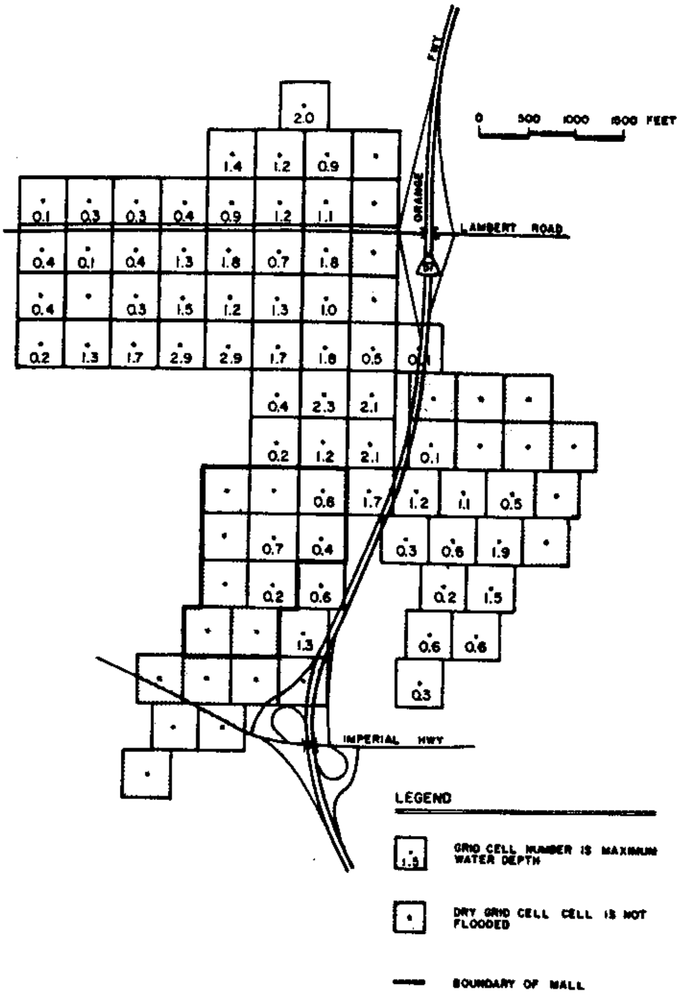

Figure 7. Maximum Flood Depths for 20-Minute Breaching Period.

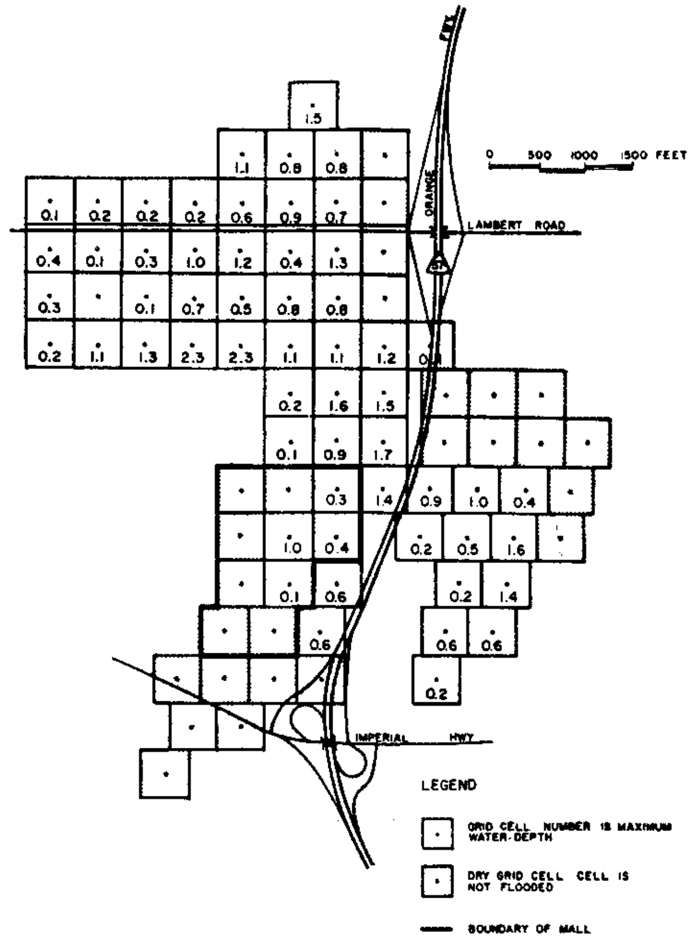

Figure 8. Maximum Flood Depths for 40-Minute Breaching Period.

CONCLUSIONS

A two-dimensional dam-break model can be used to predict flooding from the failure of a dam. The two-dimensional analysis provides a better prediction of flooding on a flat flood plain than does a one-dimensional analysis. For Orange County Reservoir, the two-dimensional model predicts a significantly wider inundated area than does a one-dimensional analysis because the two-dimensional model better represents the hydrodynamics of the actual field situation.

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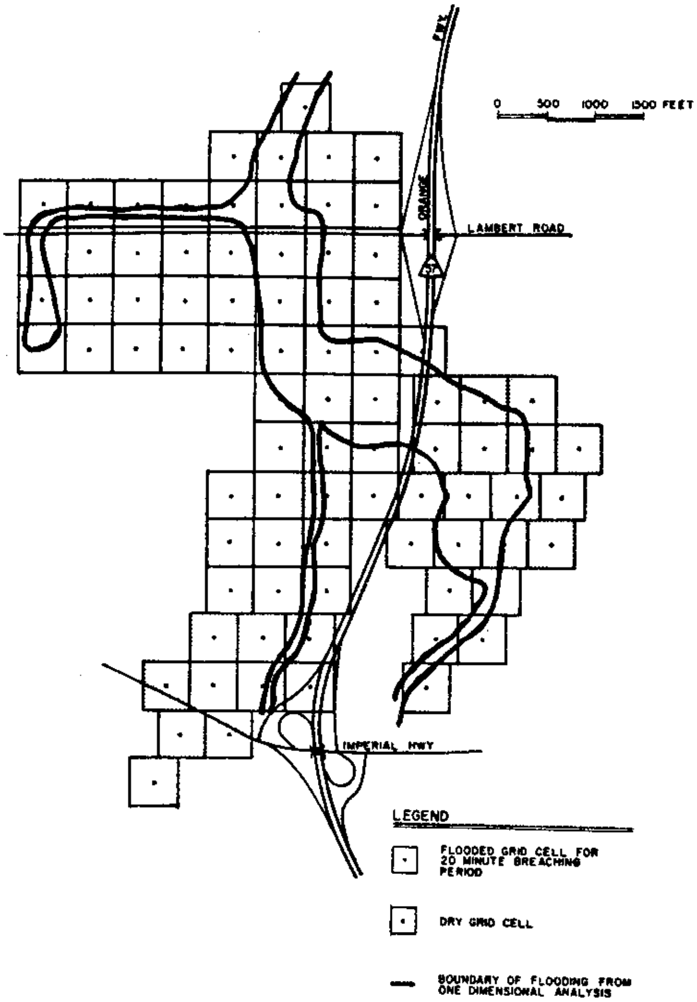

Figure 9. Predicted Inundated Areas for One- and Two-Dimensional Dam-Break Analyses